Intraparty Factions and Interparty Polarization

By

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Introduction

The issue of factions in American political life is almost as old as the Union itself. In fact, Hamilton and Madison discuss the issue in *The Federalist Papers* ("The Federalist Papers No. 9"; "The Federalist Papers No. 10"). Modern day party factions are described as "parties" and "wings." These subgroups seem to be increasingly competitive within their own parties; for example, within the Republican controlled house, the Tea Party went head-to-head with Speaker Boehner, a non-Tea Party Republican. Boehner pushed back on Tea Party attempts to shut down the government over Planned Parenthood spending. This event would contribute to Boehner's resignation of his House seat and speakership in 2015. Another issue becoming increasingly salient in American politics is that of polarization. Many data show that America has seen an unchecked growth in polarization since the 1940's, reaching what some have termed "peak polarization," (Drutman, 2016). Koger, Masket and Noel (2009) found that there is no information sharing between formal Democratic and Republican party organizations and only 15 transfers of information between Republican and Democratic groups out of a possible 518 when looking at "extended party networks." They also found factionalization within both the Democratic and Republican parties, with stronger factions on the GOP side (Koger, Masket and Noel, 2009). Indeed, polarization and factionalization seem to grow together.

My research agenda is to examine the relationship between sub-party factions and polarization in order to develop a model that relates the two. Do factions contribute to policy polarization? And if they do, what is the mechanism by which this occurs?

In his seminal work, *An Economic Theory of Democracy*, Anthony Downs (1957) develops a model of political competition and voting based on Harold Hotelling's (1929) principle of minimum differentiation. Taken together (and with Duncan Black's work) we arrive

at the median voter theorem which says that a majority rule voting system should lead to the outcome most preferred by the median voter.

However, contemporary politics forces us to reconsider this outcome. If the median voter theorem holds, why is there not a single congressional Democrat who is more conservative than a congressional Republican, and vice-versa? Why have elections consistently become less competitive? Polarization has led to a less productive political system, marked by gridlock and brinksmanship. And indeed, party factions have left both the Democratic and Republican Parties divided and less effective. This research project will offer new insights into polarization, factions, and their relationship.

Literature Review

Why It Matters

Previous academic work on polarization has centered on what are typically called "mass" or "elite" explanations—polarization driven either by the mass population or by the elite political leaders. Other scholars have studied the question in terms of institutional causes versus non-institutional causes, such as gerrymandering or income inequality, respectively. Still other scholars have looked at the "type" of polarization. This newest wave of polarization seems to be primarily issue and ideology based, whereas the polarization of the 1940s and 1950s had little basis in ideology or issue specific questions (Bafumi & Shapiro 2009).

Studies on the effect of polarization have looked at the impact it has on lawmaking and the efficiency of the government, as well as the consequences that polarization has on the economy. Economist Marina Azzimonti constructed a high frequency measure of political polarization (instead of the biannual measures usually used by political scientists) to study the economic impact of political polarization. She found that political polarization negatively affects

employment, investment and economic output. Indeed, political polarization played a role in slowing the country's economic recovery after the great recession (Azzimonti 2013).

Another area where the effects of polarization are seen is in redistricting. Partisan redistricting, or gerrymandering, can have different effects. Of course, maps may be redrawn to try to favor one party over the other. However, other more subtle effects may also come about. Redistricting can lead to instability and uncertainty—these effects may be seen as "good" since they lead to more competitive elections. However, this instability, according to scholars, can also make it more difficult for representatives to represent their constituents well. Stable elections contribute to greater knowledge of constituents' wishes among representatives, as well as constituents being better able to hold their representatives accountable (Yoshinaka & Murphy 2011).

Polarization may also lead to gridlock. Indeed, a study carried out using data from 1975-1998 (before the recent spike in polarization) found that both divided government and polarization lead to gridlock. Even when government is unified, high levels of polarization can stall productivity (the current American political experience bears this out), whereas divided but less polarized government may face less gridlock. The exception to polarization stalling productivity is when a unified government is veto- and filibuster-proof (Jones 2001).

Importantly, many of these literatures have thought of polarization as what I call a "non-discriminating" phenomenon, meaning the Democratic and Republican parties have diverging ideologies and hence polarization has increased. The intraparty/interparty dynamic has largely been ignored. Indeed, in some ways, the emergence of factions within the Democratic and Republican parties can almost be thought of as "mini" polarization within the parties. As differences within the party increase, the interparty dynamic also changes. However, this effect

need not be equal. If one party factionalizes substantially while another does not, it seems reasonable to assume that polarization will increase, even if only one party is driving the change. Of course, depending on the policy preferences of the different factions, polarization could actually decrease. Hence, considering the effect of intraparty changes on interparty dynamics offers new insights into the causes of polarization as well as the effects of factionalization.

Other Explanations of Polarization

Much time and ink has been devoted to studying the phenomenon of polarization in the United States. Fiorina and Abrams (2008) studied political polarization in the mass American public. Contrary to typical findings, they argue that the American public has not polarized significantly and that studies of polarization have been the plagued by "misinterpretations and misconceptions." They argue that the American public has undergone sorting within subgroups. Essentially, their argument is that the vast majority of Americans are moderate, while elites drive polarization. Abramowitz and Saunders (2008) are critical of this hypothesis. They show, using different data, that in fact only the least politically active members of the public are in the middle of the political spectrum. According to these authors, politically active members of the public (and the elite) have definitely polarized.

Another given cause of polarization is the media (which could be argued is an "elite" explanation of polarization, although not in the typical "political elite" sense). This argument posits that an increasingly partisan and polarized media has driven the polarization of the electorate in this country. The story goes that partisan media emerging after the 1970's has been able to persuade voters to have more partisan and hence polarized views (Prior 2013). However, there are some problems with this argument. One counter-argument is that partisan voters existed prior to the emergence of highly partisan media and that once this media emerged these already

partisan viewers were drawn to it. This, then, does not represent the media polarizing voters, but rather the media changing its coverage to attract more existing viewers. Further, studies have shown that there are a number of problems in surveys where people self-report their media use. Indeed, a review of literature on media and polarization found that although media audiences did migrate when more partisan media became available, research has not supported the idea that partisan media has led to a more polarized public (Prior 2013).

Other studies have considered alternative causes of polarization. A study on the effect of gerrymandering found that the practice has little effect on polarization—although the authors did observe the same sorting effect that Fiorina described (McCarty, Poole & Rosenthal 2009). The same authors also investigated income inequality as a cause of polarization. Indeed, they found that relative income is a strong predictor of party identification and increasing income inequality can help explain polarization (McCarty, Poole & Rosenthal 2003). Finally, another interesting explanation is that governmental failures can explain polarization. When policies fail, the two main reactions are either "this policy was doomed to fail, let's stop it" or "this policy can work, we just didn't use a strong enough form." These opposite reactions lead to a divergence in voter opinion and hence polarization (Dixit and Weibull 2007).

Taken together the conclusion can be that experts are not entirely sure what causes polarization. Indeed, it is likely a combination of different groups and mechanisms—if income inequality is an explanation, then mass driven polarization is likely the mechanism. If gerrymandering turns out to be a large factor in polarization, then perhaps elite explanations make more sense. The dynamic between intraparty factions and interparty polarization needs to be explored further as a cause of polarization. If large, influential party factions can pull the

ideological median of a party to the left or right then it makes sense that the factions can be given as a cause of polarization.

What We Know about Intraparty Factions

Intraparty factions are not new phenomena. Indeed, party factions have existed in the United States since the development of the Republic (Sin 2014). These factions vary in size and importance, with certain groups dominating parties at times and multiple groups sharing power at others. Sin (2014) shows that since 1879, in general, two major intraparty groups have existed in each party.

Much political science literature has focused on two levels—the individual and the party. The individual can take the form of anything from a typical voting citizen to a lawmaker in congress and, in the United States, the party is either the Democratic or the Republican Party. Between these levels exists the intraparty factions that have, as Roemer (2004) notes, received comparably much less attention. These factions are made up of individuals and sit within larger parties. Sin (2014) says that, "Intraparty groups consist of clusters of individuals within a party who share an ideology and a set of core policy preferences." DiSalvo (2009) offers the following definition of factions: "a party subunit that has enough ideological consistency, organizational capacity, and temporal durability to influence policy making, the party's image, and the congressional balance of power." Clearly then, the defining feature of a party faction is a shared ideology.

DiSalvo's (2009) work on factions developed not only a good definition and understanding of how they fit into parties, but also how they shape Congress. He found that rising or dominant factions (in terms of caucus numbers) attempt to centralize power in congress, while smaller factions seek to decentralize power. DiSalvo also found that factions offer a way to

channel and coordinate the interests of members, empowering them and giving them more clout than they would have as individuals. Indeed, a different study found that members of intraparty factions stick together more on ideology than they do with non-factional members of the same party (Lucas and Deutchman 2009). This allows lawmakers with similar ideological beliefs to coordinate and vote together on issues, even when those votes are against their own party.

Roemer (1999; 2001; 2004) says that three "types" of factions exist in each party: reformists, militants and opportunists. Opportunists are interested only in their party winning, reformists seek to maximize the expected utility of the party and militants are concerned only with ideology, wanting a policy proposal as close as possible to its ideal point (Roemer 2001). Roemer (2004) says that these factions within each party *bargain* over the party's policy and that parties *compete* with each other. This strategic play leads to what Roemer calls a party-unanimity Nash equilibrium (PUNE). One of the key benefits of Roemer's model is that it exists in two-dimensional policy space.

Roemer's taxonomy of factions is somewhat different from the previous explanations of factions. Sin and DiSalvo consider factions to be ideologically distinct groups within a party. Ostensibly, Sin's and DiSalvo's factions should want policy outcomes as close to their ideal point as possible. Roemer's factions have differing goals that orient their behavior. Opportunists are office motivated, caring only that their party wins the election. while militants are ideologically motivated, wanting a policy as close as possible to their party's average. Reformists, in a sense, fall in between.

What We Do Not Know

Although we have an idea of how factions and caucuses inside congress affect rule making and voting, we do not have a great understanding of how factions affect the larger

political landscape, particularly in terms of polarization. It is reasonable to think that if factions can help lawmakers move away from their parties' position on certain policies, then similar effects can be seen at the larger level. Further, if members of the general public see themselves as belonging to certain factions or groups (the Tea Party, for example), then elites can appeal to these voters and move even further from their party's platform. With this understanding of how factions operate within congress, a model of intraparty/interparty dynamics can start to be built.

The Model

The Approach

The approach I use to study polarization is adapted from the literature on political competition. I extend the concept of equilibria to capture polarized policy. More extreme policies entering the equilibrium space represents polarization in the sense that policies that are attractive to a smaller subset of voters become tenable positions for parties to put forth. This, of course, is notably different from mass polarization where a polarizing public is driving party positions. In the framework I use, polarization can come about without any change in individual voter preferences. There are valuable discussions to be had about which way of studying polarization is the most appropriate. I argue that policy equilibrium is a valuable approach to studying polarization. Although it might not explain why two neighbors won't talk to each other due to party identification, it does illuminate why and which policies are possible in the political arena, including extremely partisan (and hence, polarized) ones. Indeed, the mere possibility, let alone implementation, of extreme policy may have an endogenous effect of making the citizenry more polarized.

I follow John Roemer's work on party factions and political competition very closely, particularly his book *Political Competition: Theory and Applications* (2001). In this book (and

earlier work), Roemer develops a new equilibrium concept called party-unanimity Nash equilibrium (PUNE). Roemer developed this equilibrium concept in order to escape the nonexistence of normal Downsian or Wittman equilibrium in multiple dimensions and with uncertainty. The PUNE framework introduces sub-party factions to the model and allows for the existence of equilibria even in the multidimensional/uncertainty game. Roemer (2001, 145) states that "Researchers have responded to the nonexistence of Nash equilibrium in pure strategies in the multidimensional game in five ways: The mixed-strategy approach, the sequential game approach, the institutional approach, the uncovered set approach, [and] the cycling approach." Roemer goes on to explain why each of these approaches is unsatisfactory for studying political competition. Indeed, I agree with his assertion that, if we believe elections are simultaneous move games, none of the above approaches is satisfactory.

Although the PUNE concept was not *designed* for unidimensional games of certainty (where Downsian and Wittman equilibria do exist), it certainly still "works" and thus I adopt the concept. Since the game with factions better represents the reality of how parties are structured, this concept should provide outcomes more similar to what is actually seen in U.S. politics. The unidimensional game is just a simplification of the n-dimensional game, where n is one.

The Set-Up

The set-up of my model closely follows a number of examples given by Roemer (2001, chaps. 1 and 8). His focus, again, is multidimensional games (although he considers some unidimensional games) while mine is in one dimension. I leave the game quite general, although I will make a few specifications in order to clarify the exposition. Thus, while much of the model looks like Roemer's (2001), there are a few variations for clarity. This game represents parties competing on a policy issue where the distribution of voter types is known. To make the game

clearer I choose the policy issue to be a proportional tax rate and the voter types to be their wages. Hence, the game is only made less general in the sense that the policy space (taxation) is bounded from 0 to 1 and voter types (wages) are bounded from 0 to infinity. I assume also that this tax rate is the only issue that voters care about (this is the unidimensionality) and that voters have single-peaked preferences over the tax policy.

Assumptions.

Altogether, I assume the following. Two parties, $i = 1, 2$ (where $i = 1$ is the Right Party and $i = 2$ is the Left Party) have payoffs as a function of any number of things including their probability of winning the election, the policy put into place and/or the policy their party plays. I assume also a unidimensional policy space, over which voters have single-peaked preference orderings. Finally, I assume certainty. That is, I assume that the parties know the distribution of voter preferences (given by a probability function **F)** perfectly.

Policy.

Again, the policy over which the two parties are competing is a proportional tax rate. Since the tax rate can neither be lower than 0%, nor greater than 100%, $t \in [0, 1]$ is the policy space (unfortunately this makes for a confusing policy space, spatially). The tax collected per person is simply a product of the tax rate, t, and that person's wage, w. If we call the mean of the population wages, μ, then it can be shown that that the average tax revenue per person is $t\mu$. Namely, integrating over the wages gives the desired result,

$$\int_{w=0}^{w=\infty} twd\mathbf{F}(w) = t\mu$$

(c.f. Ortuño Ortín and Roemer 2000, 8).

Actors.

The first actor in this game is the voter. Voters have direct utility functions that capture their utility from both a private good (x) and a public good (G). This utility function is

$$u(x, G) = x + \alpha G^{\beta},\ where\ \alpha > 0\ and\ 0 < \beta < 1$$

(c.f. Ortuño Ortín and Roemer 2000, 8). The private good in this case is an individual's wage, w. Assuming that the government distributes the tax revenue equally in the form of the public good, then $G = t\mu$. Hence, the voter's indirect utility function is

$$v(t, w) = (1 - t)w + \alpha(t\mu)^{\beta}$$

which I will assume is continuous (c.f. Ortuño Ortín and Roemer 2000, 8; Roemer 2001, 18). Now, the voter's ideal policy (t_w^*, indexed for this voter's wage) can be solved for by taking a partial derivative with respect to t and setting it equal to 0, and taking the minimum of this value and 1 (since the tax rate cannot be greater than 100%), which yields,

$$t_w^* = \min[\left(\frac{w}{\alpha\beta\mu^{\beta}}\right)^{\frac{1}{\beta-1}},\ 1]$$

(c.f. Roemer 2001, 14).[1]

Other key groups of actors in this game are the factions. These are the factions that Roemer (2001, chap. 8) lays out. The first faction is the "opportunists." The members of this faction are the same characters that appear in Downs' *An Economic Theory of Democracy* (1951). Their only concern is maximizing the probability that their party wins. They are solely

[1] To further illustrate, consider an example given in Roemer 2001 (24-25) and assume the public good is a simple redistribution of the tax income. Then, $v(t, w) = (1 - t)w + t\mu$ and $t^* = 1$ for those with wages less than the mean ($w < \mu$) and $t^* = 0$ for those with a wage greater than the mean ($w > \mu$). Since in all real economies the median wage (m) is less than the mean (i.e. $m < \mu$) a tax rate of unity will always prevail in this example (or the party proposing the tax rate closest to unity will win). For this reason, I do not follow this example through the rest of the way, in order to allow for more interesting cases.

interested in winning office and they have no concern for policy. Let (t_1, t_2) be a pair of policies proposed by party one and party two and (s_1, s_2) be another pair of different policies. Then, for opportunists,

$$(t_1,\ t_2,) \succcurlyeq (s_1,\ s_2) \Leftrightarrow \pi(t_1,\ t_2) \geq \pi(s_1,\ s_2),$$

where $\pi(t_1, t_2)$ is the probability party one wins playing t_1 against t_2 and $\pi(s_1, s_2)$ is the probability party one wins playing s_1 versus s_2 (and vice-versa for party two) (Roemer 2001, 148).

Again, continuing to follow Roemer, the second faction is the "reformists." These are the characters of Wittman's "Parties as Utility Maximizers" (1973) and "Candidate Motivation: A Synthesis of Alternative Theories" (1983). The members of this faction care only about the policy ultimately implemented by the winning party; the election/holding office is simply a means to a policy end. Of course, to get to that end, the party *does* need to win the election. Hence, the reformists maximize the party's expected utility and for two different policy pairs,

$$(t_1,\ t_2,) \succcurlyeq (s_1,\ s_2) \Leftrightarrow \pi(t_1,\ t_2)v_1(t_1) + \Big(1 - \pi(t_1,\ t_2)\Big)\, v_1(t_2)$$

$$\geq \pi(s_1,\ s_2)v_1(s_1) + \Big(1 - \pi(s_1,\ s_2)\Big)\, v_1(s_2)$$

(Roemer 2001, 148).

The third and final faction is that of the "militants." This faction will be the most important in the games and modifications presented shortly. The militant group is only interested in the policy their party proposes. That is, they do not care about winning office (as the opportunists do) or the policy actually enacted by the winning party (as the reformists do). Militants can be thought of as hardliners or purists; their utility comes entirely from a "pure" policy proposal from their party. Roemer (2001, 148) says that militants are "interested in publicity." The party adopting their ideal policy position acts as a sort of advertisement for that

ideological position. The idea is that putting forth this policy can convince some voters to shift their preferences towards that of the militants. Since militants care only about their own party's position, the other party's proposed policy has no bearing on their preference ordering, and hence

$$(t_1,\ t_2) \succcurlyeq (s_1,\ s_2) \Leftrightarrow v_1(t_1) \geq v_1(s_1)$$

(Roemer 2001, 148). It is hard to say what exactly is meant when a faction is referred to outside this framework. For example, is the Freedom Caucus a Republican reformist or militant faction? I would argue that their behavior makes them appear closer to a militant faction than a reformist faction, but arguments could be made either way. Ultimately, most examples of real-life factions will likely have a mix of strains of the different types listed above.

A party then, is simply a composition of the three factions. The party composition can be altered depending on which model is examined. For example, a game where the parties are made up only of opportunists is the typical Downsian game and a game where the parties are made up only of the reformists is a Wittman game. In these cases, the parties simply take on the functions given above. In the game with factions, the parties do not have a single function, but rather, take into account the different factions in order to create an ordering over policies. So, a party is said to (weakly) prefer (t_1, t_2) to (s_1, s_2) if and only if all factions weakly prefer (t_1, t_2) to (s_1, s_2) (and for strict preference, at least one faction must strictly prefer (t_1, t_2) to (s_1, s_2); Roemer 2001, 149). Due to this construction, the parties' preference orderings (call $\prod_i$ where i indexes the parties) will be incomplete. There will be many policy pairs where two factions prefer one policy, while the other faction prefers the other policy (Roemer 2001, 149).

Party-Unanimity Nash Equilibrium (PUNE)

The concept of a "party-unanimity Nash equilibrium" (PUNE) will be vitally important to the games presented in the next section and so I define the concept. The definition of a PUNE for

a pair of policies (t_1, t_2) is that the policy pair be a Nash equilibrium for the game with $\prod_1$ and $\prod_2$ and T. In other words $(t_1, t_2) \prod_1 (s, t_2)$ (read: Party 1 prefers (t_1, t_2) to (s, t_2)) and $(t_1, t_2) \prod_2 (t_1, s)$ where $s \in T$ (Roemer 2001, 149). Essentially, a policy pair satisfies the PUNE criteria if (and only if) neither party can unanimously agree to alter its proposal, holding the other party's proposal fixed. To deviate from a given policy pair, one party's factions must be at least indifferent between the old policy and the deviation, and one faction must prefer the deviation (again, holding the other party's proposal fixed). If this condition holds, then (and only then) will the party deviate, and hence the previous policy pair was not a PUNE (Roemer 2001, 149).

The Games

With the preliminaries now in place, different games with varying factions can be examined using the PUNE concept. In this section, I set up a few different games (where parties are arrayed in different spatial manners) and I find different equilibria. I also examine what happens when different factional groups emerge within a party in the different games.

Types of Games to Be Analyzed.

For each individual game, I will solve for three different specifications. First, I will find the Downs equilibrium, which is the game where both parties are made up only of opportunists, and thus,

$$\prod_1(t_1, t_2) = \pi(t_1,\ t_2) \text{ and } \prod_2(t_1,\ t_2) = 1 - \pi(t_1, t_2).$$

The Downs equilibrium will appear as a red diamond in the figures. The second specification I will solve for is the Wittman game. This is the game where parties are made up only of reformists, and thus,

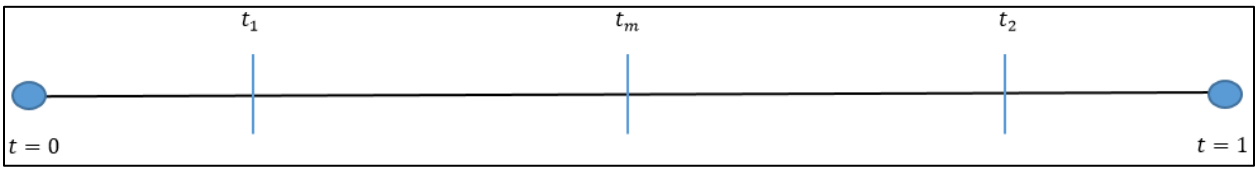
$$\prod_1(t_1, t_2) = \pi(t_1,\ t_2)v_1(t_1) + \left(1 - \pi(t_1,\ t_2)\right) v_1(t_2) \text{ and } \prod_2(t_1,\ t_2) = \pi(t_1,\ t_2)v_2(t_1) + \left(1 - \pi(t_1,\ t_2)\right) v_2(t_2).$$

Wittman equilibria will appear as a green square. Where there is a continuum of Wittman equilibria, a green line will connect the endpoints. Finally, I will consider the game where there are intraparty factions. In this game I will solve for the PUNEs (recall, parties do not have preference functions, but rather incomplete preference relations). PUNEs will always be a continuum in the unidimensional case. The continuum's endpoints will be denoted with a purple circle, and the rest of the continuum will be connected with a line.

Game with Left and Right Parties.

The first scenario I consider is the most common of games, a game with a Left Party and a Right Party. In this game, one party prefers policies left of the median (or represents voters who prefer policies left of the median) and one party prefers policies right of the median (or represents voters who prefer policies right of the median). Recalling that the policy space under consideration is that of taxation, the Left Party actually sits *to the right* of the median spatially (tax policies closer to unity) while the Right Party sits *to the left* of the median spatially (tax policies closer to zero). Denote the median voter's preferred policy t_m and the ideal policy for the Right Party by t_1 and the ideal policy for the Left Party by t_2. This "ideal policy" can either be thought of as simply a policy that the party agrees is its ideal point, or, this point can be thought of as being endogenously chosen by the members of that party (the median voter within

the Right Party has an ideal point of t_1). Thus, $t_1 < t_m < t_2$ describes the spatial ordering of the preferences. This scenario looks as such:

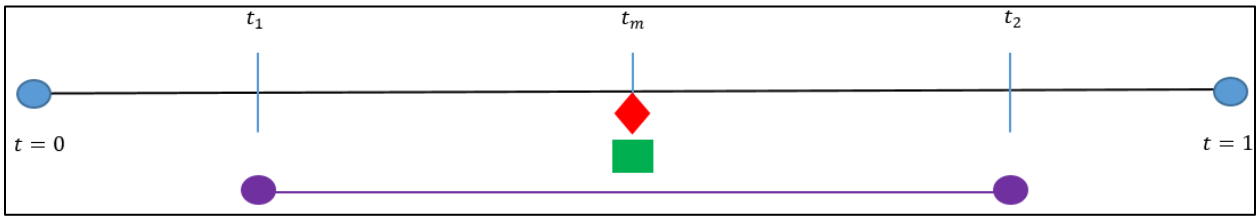


Consider first the game with only opportunists. Again, this is the Downs game, where parties compete only to win office. The well-known result is that both parties will propose the median voter's ideal policy. With the assumption that the indirect utility function of the voters is continuous and single peaked and with an assumption that no voter is indifferent between two non-identical policies this result can be proved (see Roemer 2001, 21-22) Neither party can profitably deviate, as an ε deviation right or left will result in the party going from winning with a 0.5 probability to a $\frac{1-\varepsilon}{2}$ probability of winning. Thus, t_m is the Nash equilibrium and the winning policy in this game.

Next, consider the game with only reformists. This is the Wittman game. In the unidimensional game with certainty, where both parties are comprised only of reformists, the result is again a Nash equilibrium at t_m. This can be proved if the assumptions used in the Downs game hold and the additional assumptions of monotonicity of preferences and that the fraction of voters whose ideal policy is less than some t is continuous and strictly increasing on T (Roemer 2001, 29). With these reasonable assumptions, it can again be shown that (t_m, t_m) is the Nash equilibrium (see Roemer 2001, 30-33). The intuition behind this result is straightforward. Suppose the Right Party is to the right of the median and the Left Party is to the left of the median such that $\pi(t_1,\ t_2) = 0.5$. Then both parties win with their favorite policy one-half of the time and the other party wins and gets their policy the other half of the time. But,

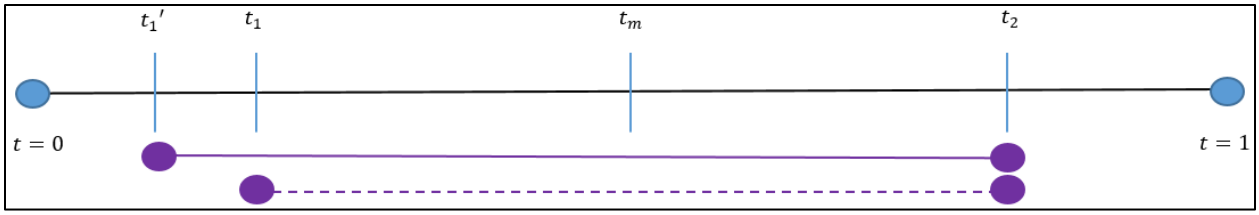
an ε deviation towards the median for any party would result in their probability of winning going to one, and they would get their favorite policy (plus or less ε) 100% of the time. Thus, these small shifts towards the median are profitable deviations until both parties reach the median.

And, finally, the game with three factions. It turns out that given the unanimous structure of PUNE, the equilibria are somewhat uninteresting. Call t_1 the policy preferred by the Right Party's factions and t_2 the policy preferred by the Left Party's militants. The PUNEs for this game are simply the continuum from t_1 to t_2. This is because, for all policies between t_1 and t_m the Right Party's militant factions have preferences opposed to the opportunists and reformists. The militants constantly want the party to move further to the right on policy (to the left spatially) towards their ideal point while the opportunists and reformists want the party to move to the center to maximize the party's chances of winning. This exact same conflict of preferences exists in the Left Party and hence the policies between t_m and t_2 are in the continuum of equilibria. The Downs equilibrium, Wittman equilibrium and PUNE set are shown below (recall, the Downs equilibrium is a red diamond, the Wittman equilibrium is a green square and the PUNE set is a line showing the continuum with purple circle endpoints):



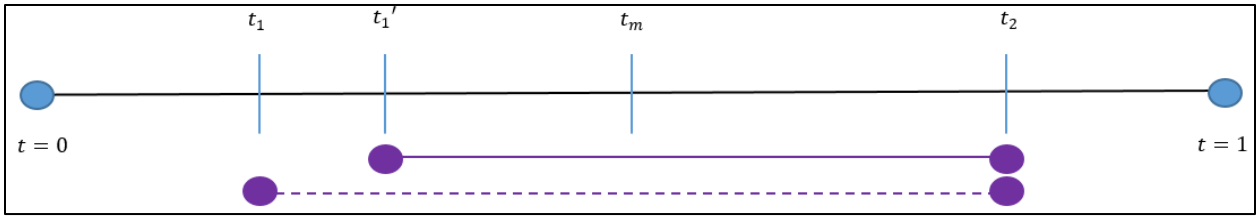
Now consider only the game with all three factions. What will happen if a new faction emerges in either party? Well, first, since the opportunist and reformist factions have identical ideal policies of t_m, a new opportunist or reformist faction for either party will have no effect on

the PUNEs and these new factions will be indistinguishable from the other opportunists/reformists. A new militant faction, however, is more interesting. For clarity, I show only the effect of one party's militants changing, although the intuition holds for both parties. First, consider the emergence of a new, more extreme militant faction in the Right Party. This group's ideal policy is t_1', while the old militant faction's ideal policy remains at t_1. The new continuum is the solid line, while the old continuum is the dashed line:



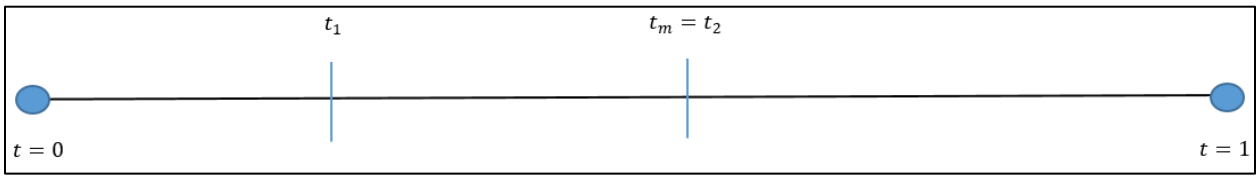
The PUNEs in this game *are the same* whether the old militants (at t_1) continue to exist or not. The diagram without the t_1 militants looks the same as the above figure except without the vertical tick mark denoting the t_1 militants' ideal policy.

Now consider the emergence of a more moderate militant faction in the Right Party. Again, this new faction has an ideal policy denoted by t_1'. In this case, it does in fact matter if the old militant faction at t_1 continues to exist. If the old t_1 militant faction itself moves to t_1' (and so it is not really a "new" faction, but rather the old faction with new preferences) or dissolves after the emergence of a new militant faction at t_1', then the PUNE set changes. If the old militant faction continues to exist at t_1 then the PUNE set does not change. The PUNE set where the old faction no longer exists is represented with a solid line, while the PUNE set where the old faction continues to exist is represented with a dashed line:



Game with Left Party at Median and Right Party.

The second game changes the placement of the parties slightly. In this game, the Left Party sits exactly at the median (or, in the game with factions, the left's militants are at exactly the median) while the Right Party sits to the right (left spatially) of the median. So, in this version, $t_1 < t_m = t_2$, which looks like:

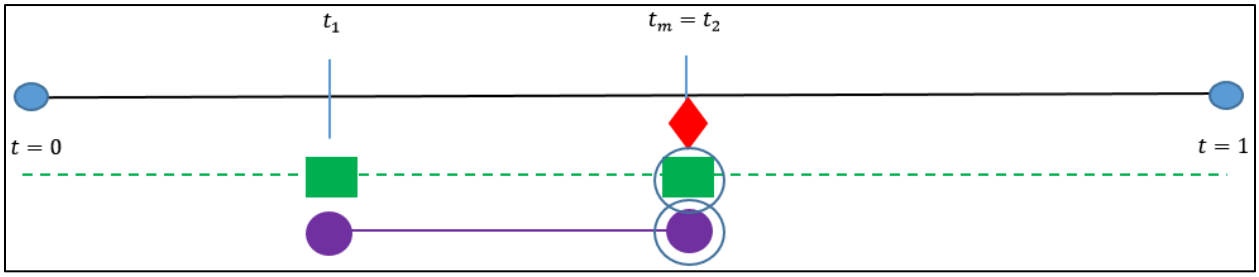


Starting again with the Downs/opportunist game, the Nash equilibrium is found to be the median (and the Left Party's ideal point). Recalling the assumptions made in the first (Left-Right) game, this result can be shown true. The exact same logic holds. Each party is composed only of opportunist factions, and hence each party cares only about maximizing their probability of winning, which means playing the median voter's ideal policy point, and winning office (with policy $t_m = t_2$) with a one-half chance.

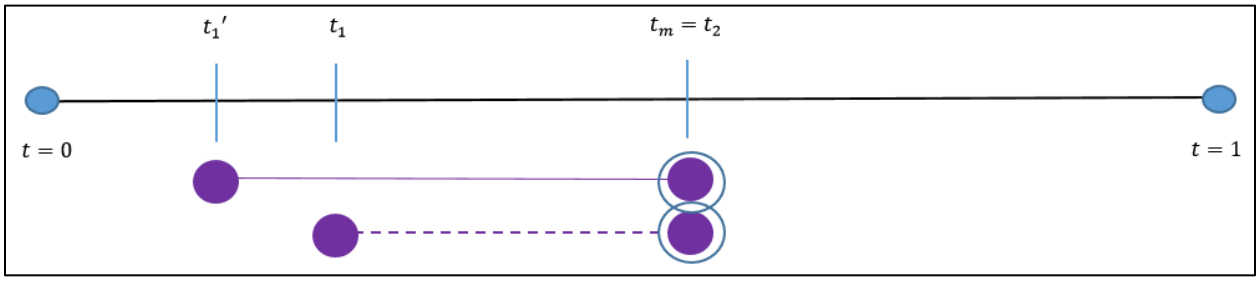
The Wittman/reformist game's equilibrium, however, differs from the result found in the Left-Right game. One assumption necessary to obtain a unique Wittman equilibrium is having $t_1 < t_m < t_2$ (Roemer 2001, 29). Since in this game $t_m = t_2$ the condition of $t_m < t_2$ does not hold. Since both parties know the distribution of voter preferences, they know that when this

game is played $\pi(t_1,\ t_2 = t_m) = 0$ unless $t_1 = t_m = t_2$; so, $t_m = t_2$ will always win. In theory, the Right Party could play any policy because its utility will be $v_1(t_2 = t_m)$ regardless. Therefore, the policy pairs that can be played are, theoretically, the entire policy space, but $t_2 = t_m$ will always win. It seems reasonable to assume that, at the very least, the Right Party will not play a policy to the left of t_m. It seems though, that the Right Party is more likely to play either t_1 (its ideal policy) or t_m (the winning policy).

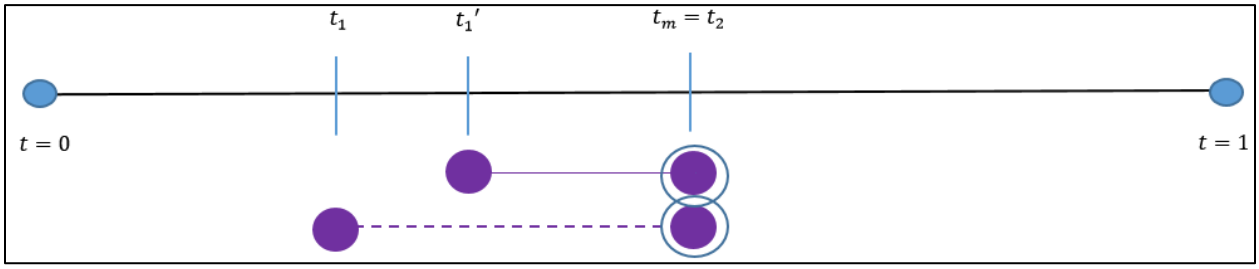
The game with factions allows us to escape this odd result. The PUNEs will span from t_1 to $t_m = t_2$. This narrows the policies played by the Right Party from *all of them* to a more reasonable continuum. The presence of the militant faction in the Right Party will eliminate tax rates less than t_1 from being proposed. The Right Party's opportunists prefer t_m be played and thus the policies in the region between t_1 and t_m may be proposed. All of the Left Party's factions actually agree on playing $t_2 = t_m$ since this is the ideal policy for the militants and the reformists, and results in the highest probability of winning, satisfying the opportunists. The Downs equilibrium, Wittman equilibrium and PUNEs are shown below (recall, the Downs equilibrium is a red diamond, the Wittman equilibria appear as a green dotted line while the green squares represent the Right Party's and Left Party's ideal polices and the PUNEs are purple circles at the endpoints with a line showing the continuum. The winning policies for the Wittman and PUNE are circled):



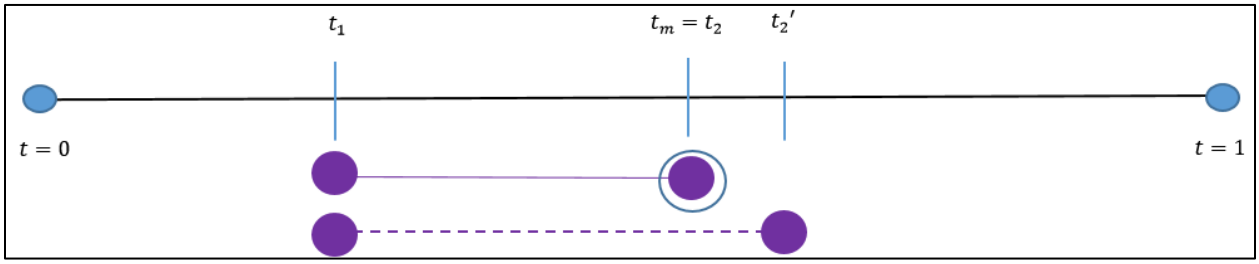
If new factions emerge, the result is very similar to that of the Left-Right game. If a new, more radical militant group forms in the Right Party, then the PUNE space will expand further to the right (left spatially). Again, this "emergence" could be either the existing militant group becoming more radical, or an entirely new group forming while the old militants remain. In either case, the PUNE set expands in the same way. Call again the new militant faction's ideal policy t_1', I depict the new PUNE space with a solid line in the figure below, while the original PUNE set exists in the space of the dashed line:



The emergence of a more moderate militant faction in the Right Party leads to exactly the same situation as before. If this new militant faction coincides with either the dissolution of the old, more extreme militant faction, or the moving of the old militant faction to the new group's position, then the PUNE space shrinks (the solid line case in the figure). If, on the other hand, the old (more rightist) militant faction continues to exist then the new, more moderate militant faction has no effect on the PUNE set (the dashed line):



The case with the deviation of the Left Party's factions is much more interesting. In the first case, if they deviate to the left (spatially to the right) then the game breaks down into the Left-Right game. Additionally, the known outcome/winning policy (namely, $t_m = t_2$) no longer exists. Of course, all policy pairs within the PUNE set continue to potentially be proposed. This scenario looks as such, with the original scenario having a solid line and the version where a more radical Left militant group emerges taking the dashed line:

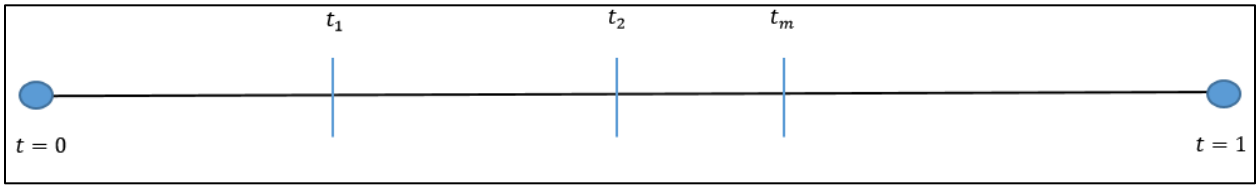


Finally, there is the case where the Left Party's militants deviate to the right (spatially left) of the median. I omit depicting or discussing this case in a separate diagram as it will be the last and final game set-up discussed in this section.

Game with Left and Right Party on the Same Side of the Median.

The last game has both the Left Party and Right Party sitting on the same side of the median. This is not how one normally thinks of parties being arrayed. The normal depiction is the Left-Right game where parties sit on either side of the median and argue for their ideological position. Recalling footnote one, however, this set-up makes sense for certain policies. As is worked through in the footnote, if the public good is a simple redistribution of the tax, and the median wage is less than the average wage, then the winning policy should always be a tax rate of unity (entire redistribution of wealth). However, consider that the Right Party represents the *super wealthy* and the Left Party represents merely the *wealthy*. In this case, both parties would propose tax rates less than the median voter's preferred tax rate! This is as far as I will follow

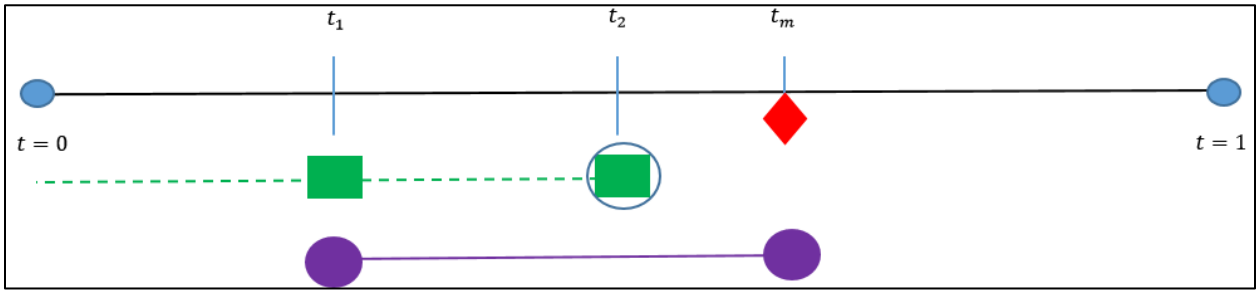
this example here (I want to leave open the possibility of voters who favor tax rates higher than the median voter, and this example does not allow for this), but it is a good illustration of how this arrangement of parties could come about. This arrangement (where $t_m < 1$) is depicted below.



For one last time, the Downs/opportunist equilibrium again resides at the median voter's preferred policy. Recalling the assumptions and logic of the previous games with only the office-seeking players, this result can again be shown true. Playing the median voter's favorite policy will result in a one-half chance of winning, while playing any other policy gives no chance of winning office.

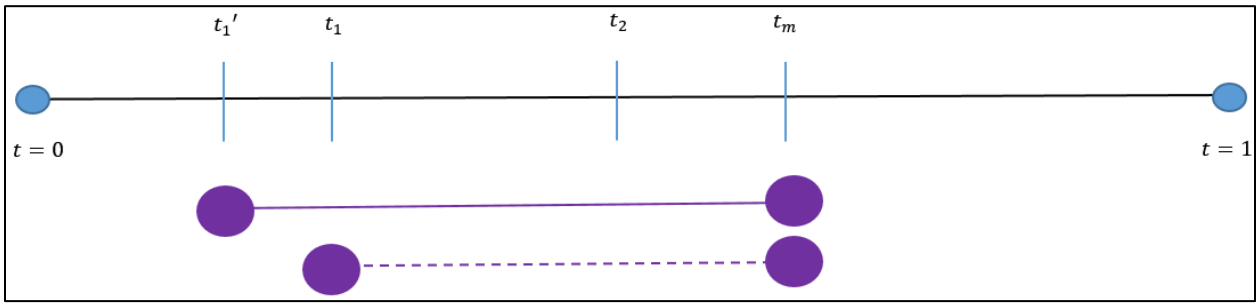
The Wittman/reformist equilibrium is more interesting in this set-up than the previous game where the Left Party was at the median. Since the Left Party's ideal policy is closer to the Right Party's ideal policy than the median voter's preferred policy, the Right Party will not play anything to the right (spatially left) of the Left Party's ideal policy. Thus, the condition where any policy being played by the Right Party in equilibrium is avoided. The Right Party cannot do any better than the Left Party's ideal policy and can only do worse if it plays a policy more leftist than that of the Left Party. The Left Party cannot do any better than to simply propose its ideal policy. Hence, the Wittman equilibria is bounded between $t = 0$ and t_2 and policy t_2 will always be the enacted policy (the Left Party will win with probability one if the Right Party plays anything else, or the Right Party can simply play t_2 as well and each party wins with probability one-half, but t_2 is enacted regardless).

Lastly, the game with all three factions. The PUNE set for this game is interesting. Although both parties' factions prefer tax rates lower than the median voter's ideal policy, all policies up to the median voter's ideal point are included in the PUNE set. This is because of the presence of opportunists. The PUNE concept does cut off any rates less than t_1, unlike the Wittman game, because the opportunist and militant factions of the Right Party prefer t_1 while the reformists are indifferent. In this game, there is no policy that will be enacted for sure. Instead, whichever policy is proposed that is closest to the median will win, but there is no guarantee that either party will, for instance, propose t_m. The Downs equilibrium, Wittman equilibria and PUNE set are shown below (recall, the Downs equilibrium is a red diamond, the Wittman equilibria appear as a green dotted line while the green squares represent the Right Party's and Left Party's ideal polices and the PUNE set are purple circles at the endpoints with a line showing the continuum. The winning policy for the Wittman equilibria is circled):

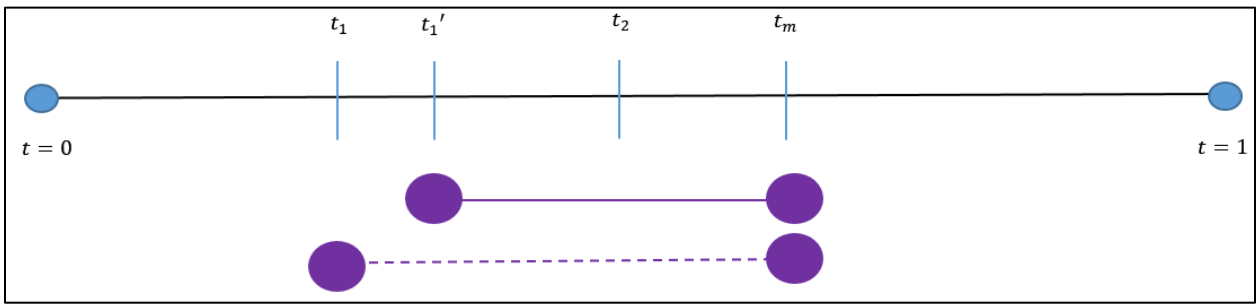


The shifting of factions in this game looks the same as in the previous games. If the Right Party has a new, more extreme faction emerge (whether the old faction becomes more extreme or an entirely new group forms) the PUNE set will expand closer to $t = 0$. The new faction's policy

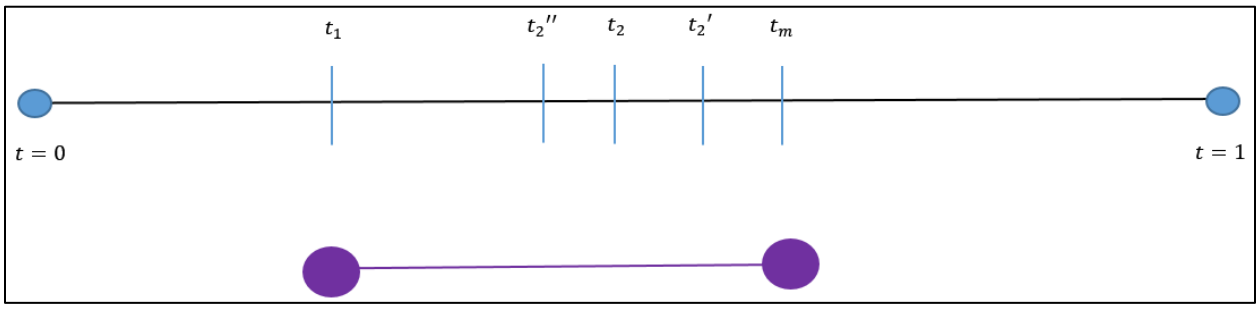
is t_1' and the new PUNE set is represented with the solid line, the old equilibria are represented with a dashed line:



The emergence of a new, more moderate militant faction in the Right Party (again with a policy of t_1') results in the same shifts as before. If the old militant faction shifts to this new position, or if the old militant faction dissolves after the new faction emerges, then the PUNE space shrinks. If the old militant faction continues to exist at its old position then the PUNE set does not change. Below, the PUNE set where the old militant factions shift or dissolve are given with the solid line, while the PUNE set when the old militant faction stays at its t_1 position is given with the dashed line:



The Left Party is a different situation. In the Left Party, the "most extreme" (in terms of leftist policy) faction is the opportunists. Moreover, since the Right Party prefers policies to the right of the Left Party's militants, new Left militants will actually not shift the PUNE space. I show both of these "cases" in the figure below with t_2' being the new, more leftist faction and t_2'' being a new Left Party militant faction with more rightist policies. In either case the PUNE set does not change.



The Left Party's militants shifting only "matters" if they shift as far as the median (in which case the game changes to the game with a Right Party and a Left Party at the median) or if they shift beyond the median to the left (in which case the game reverts to the Left-Right game).

A Brief Extension: Primary Elections

I present now a brief extension of the factional game—a version where both parties first have primary elections before moving into a general election. To my knowledge, neither Roemer nor any other researcher has extended the PUNE/factional model to primary elections. Two preliminary approaches are presented in this section, and then the framework of what might be a more satisfactory (and realistic) approach is presented in the "further work" section. The first stage is an election amongst the candidates (and factions) to determine who will represent the party. The second stage of the game is competition between the parties (and their factions) in the general election. Now, this set-up likely only works when we consider that there are, for

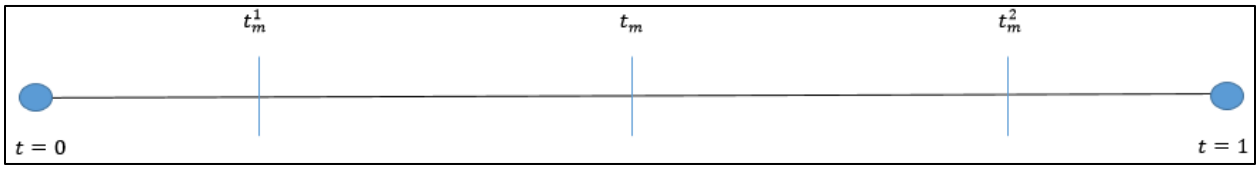
example, many elections for House seats. When these primaries are aggregated, all of the factions will be present in the greater "competition" between the two parties in the general election (not in any one election, but in the bigger picture as a whole). This framework might not fit as well for the Presidential election, however, because only one candidate wins from each party and so the factional type is "decided" after the primary. However, even the Presidential election might fit this model, because prior to the primaries there is competition between parties on which policies are the best for the country.

Modifications and Assumptions

Regardless, I proceed to present the extension. Assume now that the set of voters the Right Party represents have a median voter with ideal point t_m^1 and the set of voters the Left Party represents have a median voter with ideal point t_m^2. Assume further that

$$t_m^1 < t_m < t_m^2$$

(where t_m is the median of the entire set of voters) and thus these are primaries before the Left-Right game presented above. This arrangement is presented below:



For the primary game, I drop the reformists so that the competition is only between the opportunists and reformists (an explanation for why is provided in footnote two on page 29; a possible way of proceeding without dropping any factions is presented in the "further work" section). Let the policy the opportunists play in the primary be t_i^O (where $i = 1, 2$ for the Right and Left Parties, respectively), and the policy the militants play be t_i^M. *The factions are also*

allowed to change their played policies between the primary and general election (a potential version where this is not the case is presented in "further work").

I also make a slight modification to the opportunists' utility functions. For the opportunists, their utility function in the primary becomes:

$$\left(t_i^O,\, t_i^M\right) \succcurlyeq \left(s_i^O,\, t_i^M\right) \Leftrightarrow \pi\left(t_i^O,\, t_i^M\right) \geq \pi\left(s_i^O,\, t_i^M\right)$$

(note that the index should be the same for all of the above as factions in the same party are competing against each other in the primary election) while their utility function in the general election remains the same as before. Therefore, in the primary, the opportunists play a policy that maximizes their probability of winning the primary election and then in the general election they play a policy that maximizes their party's chances of winning the general election.

The militants' utility function is essentially the same thing, although technically it is written as:

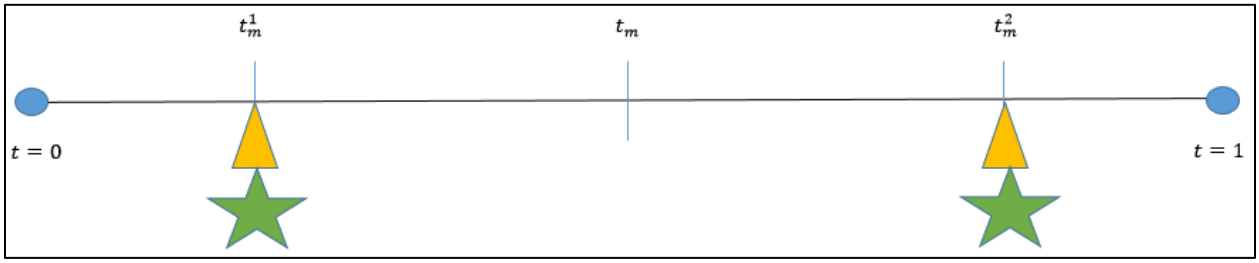
$$\left(t_i^M,\, t_i^O\right) \succcurlyeq \left(s_i^M,\, t_i^O\right) \Leftrightarrow v_i^M(t_i^M) \geq v_i^M(s_i^M)$$

(again taking the same indexes) while their utility function in the general election remains the same as before. The militants, unlike the opportunists, will play the same policy in the primary and general elections. (The only case where the opportunists would play the same policy in the primary and general election with these changed utility functions is when $t_m^1 = t_m = t_m^2$.)

Game Where Militants Represent the Median Party Voter

In the first of the two versions of this primary-general game I situate the militant factions so that their ideal point is the same as the party's median voter (they could also represent the average party member, as Roemer 2004, 17 discusses). In response, the opportunists will also play the party's median voter's ideal point and both candidates/factions will win the primary election with probability one-half. The arrangement is shown below, where the policy played by

the opportunists in the primary is a gold triangle and the policy played by the militants is a green star[2]:

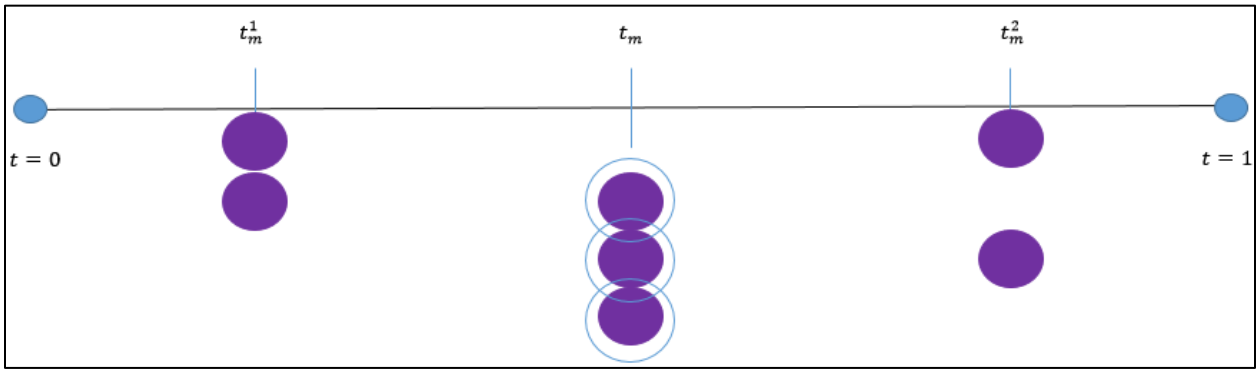


If a new faction emerges in either party and it is located at any point either than t_1^M or t_2^M it will lose the primary for sure. If the new faction it located at t_1^M or t_2^M then it is not actually any different from the existing t_i^M militants and t_i^O opportunists. I do not consider here the case where the militant faction shifts (and so has an ideal policy other than the party median voter's) but I present that as an entire separate case in the proceeding subsection.

Since the factions in both parties win with a one-half chance each, there are four possible outcomes: the Right Party's militants win and the Left Party's militants win, the Right Party's militants win and the Left Party's opportunists win, the Right Party's opportunists win and the Left Party's militants win or the Right Party's opportunists win and the Left Party's opportunists win. Instead of presenting all of these cases in separate figures, I present them all in one below. The PUNEs for the Right Party militants vs. Left Party militants, Right Party militants vs. Left Party opportunists, Right Party opportunists vs. Left Party militants and Left Party opportunists

[2] Here we can see why it is necessary to drop the reformists. Imagine that the reformists existed in this game, and that their utility function also adjusted as the opportunists' does in order to maximize the expected value of the party first in the primary election and then in the general election. Then, in order to implement the policy that will maximize the expected value of the party, they must also maximize their probability of winning. This results in the reformists and opportunists "sandwiching" the militants on either side of t_m^i in order to increase their chances of winning. In this game with all three factions, the militants will *never* win the primary election.

vs. Right Party opportunists are all presented below as purple circles, with the winning policy (if there is one) circled:



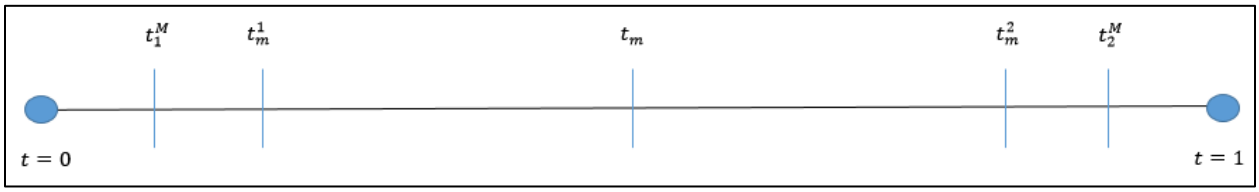
If the opportunists win the primary, they subsequently move to the population median voter, winning if they face a militant or winning with a one-half probability if facing another opportunist. If the militant faction wins the primary, they win with a one-half chance in the general election if they face a militant faction from the other party in the general election and lose for sure if they face an opportunist faction from the other party.

Now, this is for only one election. If we can think of "aggregating" these individual elections into the larger political competition between the two parties, then we can arrive back at the Left-Right game picture of before. Recall that I disposed of the possibility of militants moving from the party's median voter. This deviation would result in the game next depicted.

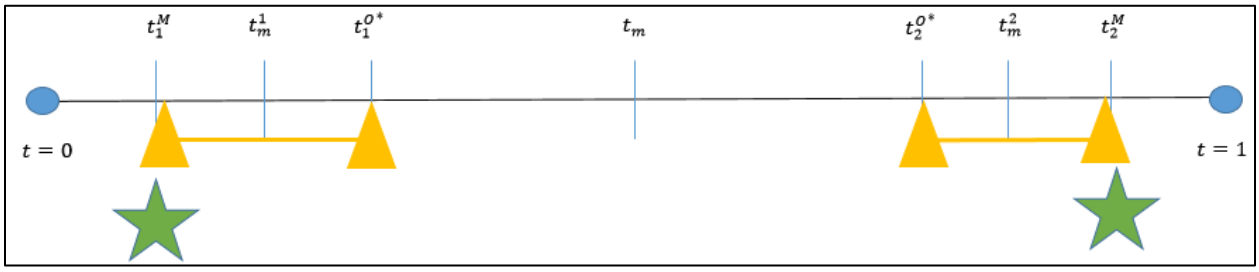
Game Where Militants Do Not Represent the Median Party Voter

If the militants do not represent the median party voter (or originally do but deviate away from the median party voter) then the opportunists win in all primaries and the general election breaks down into the Downs/opportunist game where the median voter theorem is the result. Consider that the rightist militants locate themselves at a point more right than the median Right

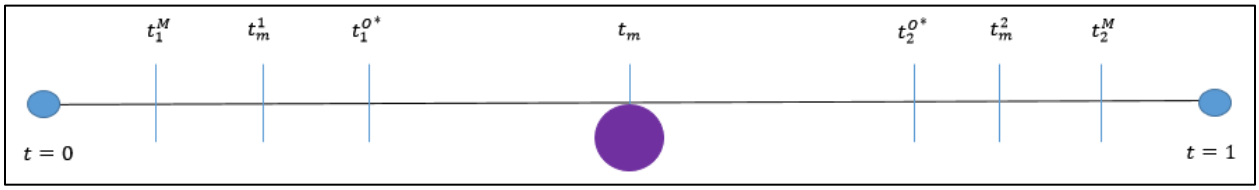
Party member ($t_1^M < t_m^1$) and the leftist militants locate themselves at a point more left than the median Left Party member ($t_2^M > t_m^2$). Then the game looks as such:



The result of this game is that the opportunists win the primary with a probability of one. In the case of the Right Party, the opportunists can locate themselves at any point to the left (right spatially) of t_1^M such that $\pi(t_1^O, t_1^M) = 1$ (and the same with the Left Party but in the opposite direction). This means that, depending how extreme the militant group is, the opportunists can locate themselves closer to the moderate side of the party median voter (ε away from the point where their probability of winning drops to one-half). This is important if things like credibility and accusations of "flip-flopping" matter in the general election. Call the point t_i^{O*} the point ε away from where the probability of opportunists winning in the primary drops from one to one-half. The policies played look as such, with the opportunists playing any policy within the continuum denote by the gold line with gold triangle endpoints and the militants playing the green star:



Therefore, it is clear that the opportunists will always win (note that the opportunists play a policy at least a shade more moderate than the militants do in order to ensure victory). This leads to a general election between opportunist factions from both parties and the median voter result, which simply looks as so (the purple circle is the PUNE/Downs equilibrium):



Of course, this game would be more interesting if factions were not allowed to move their policy positions between the primary and general elections. If this were the case then we would see both the Left and Right Parties playing something other than the median voter's preferred policy, even though the opportunists won the primary election in both cases. And, this idea of committing to one policy position is indeed quite interesting and might match what is seen in the real world better. I explore this extension in the "further work section."

Analysis

Now, with a very complete picture of the various games, equilibrium possibilities and the effects of changing militant positions, a number of observations may be made about factions and polarization. Taken together, some empirical implications may also be made.

Factions

The first and perhaps most obvious observation that can be made regarding factions is that they do, in fact, matter. In every version of the game, the PUNEs provided a different set of potential policy pairs that could be proposed when compared with the Downs and Wittman games. Indeed, only in the "Game with Left Party at Median and Right Party" did the faction

game produce a sure policy winner. And even in this game, the Right Party could propose any number of policies in equilibrium. Factional games (and PUNE) allow us to escape the idea that both parties will play the same policy (namely, the median's preferred policy) in equilibrium. The Wittman game also produces a similarly unsatisfactory result in the Left-Right game (both parties playing the median) which factions and PUNE again allows us to escape. Certainly, the large continuum that is seen in the faction game is not entirely desirable (a problem that does not exist in multidimensional games with uncertainty, see Roemer 2001) but it seems to be a closer reflection of reality than both parties proposing the same policies.

Factions and PUNE also seem to provide for more realistic results in the other two game structures (Left Party at median and both parties on one side of the median). The Wittman equilibria in the "Left Party at the Median with a Right Party" is particularly unsatisfying as, in equilibrium, the Right Party can play any policy in the policy space since it knows the Left Party will win regardless. PUNE and factions confine the policy pairs to a more reasonable space, and eliminate the possibility of the Right Party playing a policy to the left of the Left Party. In the game where the parties are on the same side of the median, factions expand the Wittman equilibrium out to the median, while eliminating policies more rightist than the Right Party's militants' ideal point. The PUNEs end up being the same as the game where the Left Party is at the median, with the exception that there is not a "guaranteed policy winner." In the game where the Left Party is at the median, the median policy will win for sure, while in the game where both parties are on one side of the median a number of different policies may win (although it is known that the policy closest to the median in the policy pair will win).

Something can also be said about the "importance" of individual factions. Generally, within each party, only two factions "matter" (see also Roemer 2001, 150). In all cases except

the case where one party is at the median (in which case all of the party's factions have the same ideal point), the two factions that matter within the party are the militants and the opportunists. The militants "pull" their party in their ideological direction while the opportunists "pull" the party towards the median. This dynamic is what produces the large continuum of equilibria. In the case where there are multiple militant groups within a party it was shown that only the "most extreme" (i.e. furthest left militant group in the Left Party and furthest right in the Right Party) matters insofar as these militants are the one setting the endpoints of the PUNEs. This result also fits nicely with Sin's (2014) finding that two major intraparty groups have existed in each U.S. national party since 1879.

One final thing can be said about factions before moving on to polarization and that is that factional strength (likely) matters. Given the way that the PUNE concept is constructed (requiring unanimity among the factions), there has been no discussion of factional strength in this study. Roemer (2001, 152) acknowledges that this is a criticism of the PUNE concept and explains how PUNE can be used as a bargaining concept (Roemer 2001, 155-158). Without going into this explanation, it is simple to think through why factional strength matters. Consider for example if the militant faction is strong within a party while the others are weak. The unanimity idea probably does not fit as well in this case. It makes sense to think that the policy the party proposes will be closer to the militant's ideal point (and the reverse is true when the opportunist or reformist (in the Left-Right game) is strongest). However, when factions have relatively equal strengths within the party, then the PUNE concept likely fits the policies that may be proposed much better. If factions are equally strong within a party, it should be expected that the party would play some policy within the PUNE space (as opposed to playing no policy and receiving the payoff of the other party's policy by default).

Polarization

Returning to the polarization piece it is clear that factions increase polarization if we think of polarization as more extreme (further from the median) policy proposals. In the Left-Right game, factions expand the policies that may be proposed (and win) significantly. The Downs and Wittman game show that only the median policy will be played and win, while the factional game includes all policies from the Right Party militants' ideal point to the Left Party militants' ideal point.

In the game where one party is at the median, the *winning* policy is not polarized (in the sense that the median policy will win in the Downs, Wittman and faction game) but there is polarization in the policies that may be *proposed*. Translating to real life, this might look like one party proposing radical, "polarized," policies, even if these policies will never win.

Finally, in the game where both parties are on one side of the median, factions have the interesting effect of making more moderate policies possible equilibria. In the game then, where both parties are on the same side of the median, the polarizing effect of factions is somewhat ambiguous. Compared to the Downs game, factions do result in polarization, while compared to the Wittman game the effect is, in fact, ambiguous. The factions eliminate the possibility of policies to the right of the Right Party being proposed, while opening up the possibility of policies closer to the median than the Left Party being proposed. If polarization is thought of slightly differently (the distance of proposed policies from the median voter's ideal point) then factions actually *reduce* polarization in this game compared to the Wittman game, as policies closer to the median enter the PUNE set.

Extension

A separate, brief analysis of the extension (with primaries) yields a few interesting results. The first falls out of the presentation of the two games—with the militants at the party median and the militants not at the party median. Anytime the militants played something other than the party median they lost. This does not necessarily mean that militants must play the party median (as nothing in their utility function says that they care about winning) but it does seem like the formulation where the militants play the median party member's ideal policy makes the most sense.

In this case, (where the militants are located at each of the party medians, respectively) a more interesting general election results, namely, a general election that is not always between only opportunists. When the militants represent the party's median voter, the militants and opportunists each win with probability one-half, leading to four different general election permutations. If these elections are "aggregated" into party competition as a whole, then the result looks like that of the Left-Right game.

What does this say about polarization? Well, in the case where the militants locate at the party's median voter it says essentially the same thing as the above section on polarization—in the Left-Right game, factions increase polarization. Even in the individual elections (not "aggregating" them into a general idea of competition) a general election between two primary-winning militant factions leads to a policy polarized general election. If militants do not locate themselves at the median, and factions are allowed to change their policy from the primary to the general election, then a primary election with extreme policy proposals is seen while the general election becomes much more moderate. If, however, factions are not allowed to move between

the primary and general election then polarized policy is seen in the general election, but not to the extent of the polarization in the normal Left-Right game.

Empirical Implications

Taken together, the above analysis leads to some empirical implications. The predictable effects of, for example, the emergence of a more extreme militant group, allow some more generalized claims to be made. I list some implications below.

1. If factions are equally strong, then polarization will be at its highest.
    a. If the militant factions are the strongest in each party, then polarization will be as high as the case where factions are equally strong.
    b. In general, if factions are weak, then polarization will be lower.
2. If a new, more extreme militant faction emerges, then polarization will increase.
3. If a new, less extreme militant faction emerges, then a decrease in polarization will only follow if the old militant faction no longer exists (if it dissolves or shifts to the new less extreme position).
4. In the case of a primary election, if a shift in policy is seen after a primary, then the dominant/winning faction of the party is likely the opportunists.

Limitations and Further Work

There are a number of limitations with the model and games presented here. These limitations are important to recognize as they have implications for the analysis of factions and polarization, as well as the empirical predictions. Much of the further work that can be done on this topic would involve eliminating some of these limitations.

Limitations

The first and most obvious limitation of this study is the specification of a unidimensional game with certainty. The idea that voters care only about one issue and that parties know voters' preferences perfectly is, of course, unrealistic. In reality, many issues are relevant to voters and there is always some uncertainty about voters including both their preferences and whether or not they will vote. A second limitation of this study is the lack of any idea of factional strength. Although some basic ideas of how factional strength may matter were presented in the analysis, a more rigorous treatment of this issue would be beneficial.

Further Work

Indeed, many of the above limitations have already been addressed. Roemer (2001 and others) presents the case of multidimensional policy spaces and uncertainty. Roemer (2001) also considers games where the parties have endogenous policies of the voters' preferences whom they represent. What hasn't been done, to my knowledge, is to extend these models further to consider issues of polarization and the effects of the emergence of new factional groups. Roemer also addresses the issue of factional strength, but this topic could be extended by considering the implications factional strength has on polarization and how emerging/dissolving factions can shift the policy space and polarization.

Another area of further work that would be fruitful is a tightening of the extension presented on primary elections. Ideally, the need to drop the reformists would be overcome and new utility functions that capture both the primary and general elections could be built for the opportunists and reformists (the militants by construction do not care about winning). Additionally, restricting or eliminating a faction's ability to move after the primary is probably a

better representation of real life and incorporates the disutility that a candidate receives from "flip-flopping" and not committing to a policy position.

Conclusion

This paper has studied the relationship between intraparty factions and polarization. A model of party competition with factions and an equilibrium concept called PUNE (both of which are due to Roemer) were used to investigate various unidimensional games with certainty where parties and factions have varying preferences. I extended Roemer's work by presenting basic spatial models to show how factions change the equilibrium space in comparison to Downs and Wittman models. Specifically, I showed how new militant factions can expand or contract the PUNE space. I then developed a brief and novel extension of the model to primary elections, showing how factions interact with multi-stage competition. Office motivated factions (and their candidates) will switch positions between the primary and general elections if they can, while militants will not. Next, I analyzed my findings and, using a policy-based concept of polarization related how factions (and changes in factions) can effect polarization. The emergence of more extreme militant groups will widen the PUNE space while the emergence of more moderate militant groups will only shrink the PUNE space if the old militant faction dissolves or moves to the new militant's point. This expanding of the PUNE space can be thought of as an increase in polarization, while the shrinking can be thought of as a decrease in polarization (compared to the original levels). In either case, the game with factions had greater levels of polarization than the game without. If individual elections can be "aggregated" to construct a larger picture of interparty competition, then the result from the primary game has the same implications as the Left-Right game in terms of polarization. Finally, I noted limitations to this study and future work that may be done.

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